

The Cosmic Coincidence Problem in the Brane World

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Abstract. The so called ‘cosmic coincidence’ problem seems to suggest a coupling between dark energy and dark matter. A possibility could be given by dark matter particles with masses depending exponentially on the scalar field associated to dark energy. If the latter also has an exponential potential, attractor solutions exist with constant Ω_{DM} and Ω_{DE} and negative effective equation of state. Due to the very low mass of the dark energy field, its coupling to baryons should be strongly suppressed. Here we present a natural realization of this scenario in a model with large extra dimensions.

1 Coincidence Problem *vs* Equation of State

One of the most puzzling aspects of the current cosmological paradigm, is the so-called ‘cosmic coincidence problem’, *i.e.* the quasi equivalence between dark matter (DM) and dark energy (DE) densities today. Indeed, for a non-interacting fluid A , the continuity equation $d(\rho_A a^3) = -p_A da^3$, relates the equation of state $w_A \equiv p_A/\rho_A$ to the evolution of the energy density with the expansion, *i.e.*, $\rho_A \sim a^{-3(w_A+1)}$. Therefore, the ratio between dark matter ($w_{DM} = 0$) and a dark-energy component with negative pressure ($w_{DE} < 0$) scales as

$$\frac{\rho_{DM}}{\rho_{DE}} \sim a^{3w_{DE}}, \quad (1)$$

and could be only ‘coincidentally’ of order one today. This is the case for the cosmological constant ($w_{DE} = -1$), for a scalar field with an inverse power law potential $V(\varphi) \sim \varphi^{-n}$ ($w_{DE} = -2/(n+2)$), or with any potential allowing solutions with negative pressure. On the other hand, exponential potentials exhibit attractor solutions in which there is no cosmic coincidence problem, since the scalar field energy density scales at a constant ratio with the background, be it matter or radiation. But, by virtue of eq. (1) this implies $w_{DE} = 0$ after equivalence, thus making it unacceptable as a DE component. Thus, there seems to be an inextricable contradiction between a negative equation of state and the elimination of the cosmic coincidence.

The impasse may be escaped by relaxing the assumption leading to eq. (1), *i.e.* by allowing an interaction between DM and DE, besides the gravitational one.

2 Exponential Vamps

An interesting possibility is to consider dark matter particles whose masses depend on the quintessence field, or VARYing Mass ParticleS (Vamps), which were considered in [1]. Assuming exponential dependencies both in the potential and in the DM particle mass,

$$V(\varphi) = V_0 \exp(\beta\varphi) , \quad M = M_0 \exp(-\lambda\varphi) , \quad (2)$$

an *attractor* solution exists on which both ρ_{Vamps} and ρ_{DE} scale as $a^{-3(W+1)}$, with

$$W = -\frac{\lambda}{\lambda + \beta} , \quad \text{and} \quad \Omega_{DE} = 1 - \Omega_{\text{Vamps}} = \frac{3 + \lambda(\lambda + \beta)}{(\beta + \lambda)^2} , \quad (3)$$

provided $\beta > \frac{-\lambda + \sqrt{\lambda^2 + 12}}{2}$, thus solving the coincidence problem with a negative effective equation of state.

It should be noted that the crucial requirement in order to get an attractor with the above properties, is to have *exponential* dependencies both in the potential and in the mass, a case not considered in refs. [1]. Moreover, the coupling λ between Vamps and φ should be $O(1)$ in order to have a sensibly negative equation of state. If also baryons were coupled so strongly with the almost massless φ , we would immediatly run into problem with the bounds coming from tests of the fifth force or the equivalence principle. In the remaining part of my talk, I will discuss how a model exhibiting these properties [2] can be formulated in scenarios with large extra dimensions [3].

Before doing that, let me mention that a similar model has been discussed recently in [4]. Those authors implement the DE-DM interaction by assuming non-vanishing divergences for the energy momentum tensors

$$(T_{DM}^{\mu\nu})_{;\mu} = -(T_{DE}^{\mu\nu})_{;\mu} = \lambda T_{DM} g^{\nu\rho} \varphi_{,\rho} , \quad (\text{Amendola } et \ al) \quad (4)$$

where T_{DM} is the trace of the *DM* energy-momentum tensor. The background behavior turns out to be the same as (3), but fluctuations evolve differently, since in the Vamps case we have

$$(T_{DM}^{\mu\nu})_{;\mu} = -(T_{DE}^{\mu\nu})_{;\mu} = \lambda T_{DM} u^\nu u^\rho \varphi_{,\rho} \quad (\text{for Vamps}), \quad (5)$$

where u^μ is the four-velocity of Vamps particles. This will be studied in a forthcoming paper.

3 Vamps and Large Extra Dimensions

In principle, brane world scenarios offer a suggestive contact with the DE problem. Indeed, extra spatial dimensions compactified to a size as large as $100\mu\text{m} - 1\text{mm}$ have been shown to be a viable possibility, provided that no

Standard Model (SM) field propagates through them [3]. It might then be tempting to attribute the observed value of the DE energy density to the Casimir energy associated to some field propagating in these large extra dimensions of size r , *i.e.*

$$\rho_{\text{Casimir}} \simeq 1/r^4. \quad (6)$$

If the radius r is stabilized, then the energy density behaves exactly as a cosmological constant, and we have made no substantial progress with respect to the cosmic coincidence problem. It seems then necessary to make the radius r dynamical, so that it may evolve on a cosmological time-scale. But this turns out to be quite dangerous. The ‘radion’ field, whose expectation value fixes r , is ultra-light ($m \sim H_0 \simeq 10^{-33}\text{eV}$) and couples to the trace of the SM energy momentum tensor with gravitational strength. It then behaves as a practically massless Brans-Dicke scalar with $O(1)$ couplings to matter, whereas present bounds are $O(10^{-3})$ [5].

However, besides large extra-dimensions in the $100\mu\text{m}$ range there could also be smaller ones, and this may solve the problem. Considering for simplicity only two compact subspaces, each characterized by a single radius, we note that the trace of the four-dimensional energy-momentum tensor couples with the total *volume* of the compact space, that is with the combination $r_s^{n_s} r_l^{n_l}$, where $r_{s,l}$ and $n_{s,l}$ are the radii and dimensionalities of the two subvolumes, respectively. If we assume a stabilization mechanism that fixes the *total volume* \mathcal{V} of the compactified manifold but *not its shape*, the potentially dangerous combination of radion fields associated with volume fluctuations is made harmless, whereas the orthogonal one, associated with shape deformation, is decoupled from normal matter and may then be ultra-light. As a consequence, r_l can grow on a cosmological time-scale (such that the associated Casimir energy $1/r_l^4$ decreases) and at the same time r_s shrinks so that \mathcal{V} keeps a fixed value.

If we now identify as DM candidate some stable state living on the $4 + n_s$ -dimensional brane, we may obtain a realization of the general Vamps model of eq. (2). Indeed, if the mass of the DM particle is inversely proportional to some power of r_s , as for a Kaluza-Klein (KK) state ($m_{DM} = m_{KK} \sim r_s^{-1}$), then its energy density scales as

$$\rho_{DM} \sim 1/(r_s a^3). \quad (7)$$

Assuming that the total volume of the compact subspace is fixed, the Casimir energy (6) and the DM energy density (7) give

$$V(\phi) \sim \exp\left(4\sqrt{\frac{n_s}{n_l \mathcal{N}}}\phi\right), \quad \text{and} \quad \rho_{DM} \sim \exp\left(-\sqrt{\frac{n_l}{n_s \mathcal{N}}}\phi_2\right) a^{-3}, \quad (8)$$

respectively, where ϕ is the canonically normalized field associated to shape deformations of the compact sub-manifold [2]. Moreover, ordinary matter such as baryons is only sensitive to the global volume of the compact manifold, not to its shape, and is thus decoupled from ϕ .

In conclusion, we have realized a Vamps model in which the DM-DE coupling is naturally $O(1)$, and the DE-baryon coupling is naturally suppressed. The parameters λ and β of eq. (2) are fixed by geometry, *i.e.* by n_l and n_s , and then so are the values of W and Ω_{DE} on the attractor.

In Tab. 1 we list the possible values of Ω_{DE} , W and of $H_0 t_0$, t_0 being the present age of the Universe. We limited the dimensionality of the compact space according to the theoretical prejudice coming from string theory, *i.e.* $n_l + n_s \leq 6$.

Table 1. Possible attractors for $n_l + n_s \leq 6$

n_s	n_l	Ω_{DE}	W	$H_0 t_0$
1 - 5	1	≤ 0.44	–	–
1	2	0.83	$-1/3$	1
2	2	0.68	-0.20	0.83
3	2	0.60	-0.14	0.78
4	2	0.56	-0.11	0.75
1-3	3	≥ 0.92	–	–
1-2	4	no attractor	–	–
1	5	no attractor	–	–

A noticeable fact about Tab. 1 is that the observed range for the dark energy density, $0.6 \lesssim \Omega_{DE} \lesssim 0.8$ uniquely selects the number of ‘large’ extra dimensions to be $n_l = 2$, the same value that is required by the totally unrelated issue of solving the hierarchy problem with ‘millimeter’ size extra dimensions [3].

A model-independent prediction of the framework outlined in this paper is the presence of extra dimensions in the $100 \mu\text{m}$ range. Whereas for the solution of the hierarchy problem discussed in [3] this value was *allowed*, the link with the cosmological expansion discussed in this paper makes it a true *prediction*. Even more interestingly, the expected value for r_l is quite close to the present bound of $200 \mu\text{m}$, obtained from measurements of the Newton’s law at small distances [6], and then will be quite soon tested experimentally.

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